THE SEMANTICS OF BASIC ARITHMETIC

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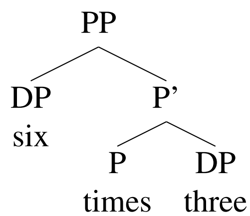
Somewhat surprisingly for such a relatively common phenomenon, neither the semantics nor the syntax of basic arithmetic expressions (things such as *seven apples plus three apples equals ten apples* or *six times four is twenty-four*) has been worked out with any real degree of rigor. To the extent that they have been examined, the typical move is to import the mathematical operations directly into the semantics, without considering the natural language implications of such a move. I will thus analyze the syntax and semantics of these expressions in terms of natural language; first I will demonstrate that syntactically these arithmetic operations are prepositions that require two arguments, and then I will move to the semantics of these operations, with a focus on multiplication. I will argue that multiplication is in fact a group-forming operation (in the sense of Landman 1989a,b), and I will also resolve the fact that *times* appears to have certain restrictions when dealing with objects instead of just numbers: the word *times* can allow (at least) one of its arguments to be a bare numeral, but it doesn't care which one (so *two apples times four is eight apples* and *two times four apples is eight apples* are equally acceptable).

In order to discuss this basic arithmetic we need to first establish their syntax. I argue (in line with people such as Pi 1999) that words like *plus*, *minus*, and *times* are prepositions. These math prepositions require both an object and a subject – in other words, two numbers or amounts to establish a relationship between. The syntax would thus be roughly as in (1):

(1)



This structure can then be used for our semantics. A semantics for something like *plus* is relatively straightforward, as it simply provides the union of the two arguments. *Times*, on the other hand, is far more intriguing; *times* requires that (at least) one of its arguments be a bare numeral, although it doesn't care which one.

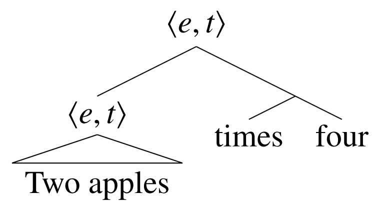
(2) a. Two apples times four is eight apples.

b. Two times four apples is eight apples.

c. ?Two apples times four apples is eight apples.

In order to analyze this construction we need to make some basic assumptions. ***apple** (where the *-operator is Link 1983's pluralization operator) is an object of type $\langle e, t\rangle$, not of type d, so we'll need a way to perform multiplication with objects instead of degrees. I propose using a group-forming operation, following Landman (1989a,b) and others, where plural objects are treated as singular. (However, for the sake of readability I will use *group* instead of Landman's ↑ notation, but the effect is the same.) This also has the advantage of making intuitive, logical sense: *two apples times four* is four groups of two apples each. So using the tree in (1), we get the denotation in (3), where ultimately x is a group of apples with two members and that are four such groups:

(3) a.



b. $[\![\text{two apples}]\!] = \lambda x.{*}\mathbf{apple}(x) \wedge |x| = 2$

c. $[\![\text{four}]\!] = \lambda x.|x| = 4$

d. $[\![\text{times}]\!] = \lambda G_{\langle e,t\rangle}\lambda F_{\langle e,t\rangle}\lambda x[group(F)(x) \wedge G(x)]$

e. $[\![\text{two apples times four}]\!] = \lambda x[group[\lambda z[{*}\mathbf{apple}(z) \wedge |z| = 2]](x) \wedge |x| = 4]$

However, note that trying to use this computation with the word order in (2b) creates an odd effect:

(4) $[\![\text{two times four apples}]\!] = \lambda x[group[\lambda z[|z| = 2]](x) \wedge [{*}\mathbf{apple}(x) \wedge |x| = 4]]$

In effect, (4) is trying to say that x is composed of 2-somes, that x is a plurality of apples, and there are four xs, but it's not clear that you can be both 2-somes and apples – can you be both a plurality and a group? An easier (though perhaps nonintuitive) way to approach the problem is to say that the constituency of (2b) is in fact *[[two times four]$_{PP}$ apples]$_{NP}$*; this makes the denotation as in (5):

(5) a. $[\![\text{two times four}]\!] = \lambda x[group[\lambda z[|z| = 2]](x) \wedge [|x| = 4]]$

b. $[\![\text{two times four apples}]\!] = \lambda x[group[\lambda z[|z| = 2]](x) \wedge [|x| = 4] \wedge {*}\mathbf{apple}(x)]$

This says that x is composed of 2-somes, that there are four xs, and that x is a plurality of apples, which is totally coherent.

It is worth taking a minute to pause and reflect upon what's happened so far. Not only have we established a simple syntax for these arithmetic operations, but we've also defined multiplication in terms of natural language, rather than being forced to insert a mathematical operation into the semantics. This allows us to create a closer link between math and language and hints at a deeper connection between the two, which is an exciting development: at least some mathematical operations are a fundamental part of natural language, and we don't need to automatically divorce mathematical language from the rest of natural language.

References

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